Cost of Capital and Corporate Refinancing Strategy: Optimization of Costs and Risks*

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Abstract

This paper investigates the effects of a firm’s refinancing policies on its cost of capital. First, existing empirical and theoretical research is utilized to assess the effect of leverage on a firm’s cost of equity and debt capital. Using these analyses, the firm’s static cost of total capital is derived, revealing an optimal financing leverage ratio as well as a penalty function quantifying the opportunity costs associated with deviations from the firm’s optimal leverage. After investigating the costs of capital restructuring, the paper then evaluates the long term effects of using dynamic refinancing strategies based upon financing leverage. Using stochastically generated cash flows and repeated simulations, refinancing boundaries are evaluated to optimize the firm’s mean financing cost, cost variability, and tail risk. By setting simple refinancing boundaries, this paper determines that a corporation can reduce average cost of capital by up to 8 percent and cost variability by up to 74 percent relative to a static financing policy.

Keywords: capital structure, refinancing policy, financial leverage, cost of capital, transaction costs

JEL Classification: C15, C61, D23, G32

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1 Introduction

1.1 Overview

Proper financing is a central problem in corporate finance, attracting considerable attention throughout academia as well as in practice. In a survey of CFO’s, Graham and Harvey (2001) found that only 19 percent of sampled firms did not have some type of target debt ratio or range. Furthermore, they found that the remaining firms varied drastically both in the flexibility of their targets as well as the sophistication of their strategies and analyses. How to set a target and, perhaps even more importantly, when to adjust the finances of the firm to reflect that target, are issues where many practitioners and academics alike disagree.

The financing mix problem originates from the unique characteristics of a firm’s capital alternatives. Corporations issuing debt benefit from a lower effective tax rate. Additionally, because of debt’s senior position in the firm’s capital structure, the risk to the investor is lower than that of equity; as a result, bond investors typically require a lower rate of return. However, debt has unique obligations because of its strongly structured terms, and higher debt levels restrict the flexibility of the firm. Additionally, high debt levels increase the cost of a potential default and bankruptcy. This increased risk translates into higher required rates of return on all sources of financing used by the firm and, hence, an increased cost of capital. This tradeoff between cost and risk implies a potential optimization where the firm reaches a minimal cost of capital and, thus, a maximal enterprise valuation.

1.2 Review of Literature

Classical theory by Modigliani and Miller (1958) forms the basis of modern capital structure analysis. In their first publication, Modigliani and Miller suggested that, under certain market conditions, the capital structure decision can be proven to be irrelevant. The assumptions for their theory were very strict and rather unrealistic, particularly:

i. No taxes exist
ii. No transaction or bankruptcy costs exist
iii. Individuals can borrow at the same interest rate as firms

Using these assumptions, Modigliani and Miller derived three propositions:
Proposition I

In Proposition I, Modigliani and Miller proved that the market value of the firm is invariant to its capital structure. In order to complete this proof, they considered a case where there are two firms A and B of equal total asset value. Firm A is financed entirely with equity and firm B uses some amount of leverage. Suppose an investor is considering the purchase of either firm A or firm B, and he may borrow debt to finance his investment. By purchasing the unlevered firm A and borrowing the same amount that firm B does, this investor is able to replicate the leveraged investment of firm B. Since the returns to both of these strategies are the same, the price of firm B must equal the price of firm A less the amount borrowed by firm B. This scenario is shown in the following figure.

![Figure 1-1: MM Proposition I](image)

Proposition II

These findings led Modigliani and Miller to another conclusion. Consider the formula for weighted average cost of capital:

\[
WACC = \frac{D}{D+E}k_d + \frac{E}{D+E}k_e
\]  \hspace{1cm} (1-1)

The first proposition tells us that \(WACC\) for the leveraged and unlevered firm must be equivalent (we may call it the asset financing cost \(k_a\)). Solving for \(k_e\), we are left with the following formula for the cost of equity:

\[
k_e = k_a + \frac{D}{E}(k_a - k_d)
\]  \hspace{1cm} (1-2)
From this result it is clear that the cost of equity must increase linearly with increases in leverage.

**Proposition III**

Modigliani and Miller’s final proposition summarized their findings: as cheaper debt replaces equity, the cost of equity increases to completely offset the savings; investment in the firm is completely unaffected by the type of securities used to finance it. For this reason, the Modigliani-Miller theorem is commonly referred to as the *irrelevance principle*.

Further generalizations by Stiglitz (1969) and others further proved that, in perfect capital markets, firm value is indeed independent of capital structure. The conditions in which these propositions hold, however, are clearly unrealistic. As Miller reflected years after the publication, while the model itself may not seem very realistic, “showing what doesn’t matter can also show, by implication, what does (1988).” Modern theory has focused on this issue.

Modigliani and Miller (1963) later released a modification to their theory removing the first assumption (no taxes exist); however, this caused a serious problem. While dividends and retained earnings are taxed under the US corporate tax system, interest payments are not. Thus, debt effectively reduces the amount of pretax income that must be paid to the government. With this benefit, it would seem that debt is a cheaper post-tax source of financing than equity at any level of leverage. The implied optimization would then be at a point where the firm is 100 percent debt financed.

Needless to say, this argument seems unrealistic. To prevent this problem, however, it is necessary that there exists an additional cost to debt such that the tax benefits of the debt are offset by the cost (Robichek and Myers 1967). Kraus and Litzenberger (1973) proposed the cost to be a risk premium, since the probability and cost of bankruptcy are greater for a highly leveraged firm. Thus, an optimal leverage ratio requires a tradeoff between the tax benefits of debt against its effect on default risk. This theory is called the *tradeoff theory*.

The theories used to analyze and explain financing behavior cover a wide spectrum from heavily quantitative to purely qualitative. Many models also appeal to current market trends and conditions to explain the refinancing decisions of firms. Some notable theories include the *pecking order theory* and *market timing hypothesis*. These situational and qualitative theories propose additional considerations which may be relevant when evaluating the results proposed in this paper.
2 Static Modeling and Optimization

2.1 Overview

This paper is divided into two broad sections: the analysis of financing leverage and cost of capital at a single point in time, and the long term optimization of financing strategy through the setting of refinancing boundary conditions. Section 2 concerns the former, deriving estimations of debt cost, equity cost, and weighted average cost of capital under different levels of static leverage.

2.2 Term Structure

Since all costs of capital will be reliant upon the term structure of interest rates, the first step in this analysis is the derivation of an interest rate model. This model will be used to forecast the rate of interest for risk free assets such as short-term US Treasury securities. The Treasury yield curve as of January 2008 is shown in Figure 2-1.

![Figure 2-1: Historical Treasury Yields](image)

To generate a continuous curve from these data, I use a fitting of the rational function with three points: the initial rate \( r_0 \), an intermediary rate 2 years forward \( r_2 \), and a terminal rate \( r_\infty \). The equation is shown below and derived in Appendix A.

\[
\mu(t) = 1 + \frac{\left( -1 + \frac{r_1}{r_0} + \Omega \ r_1 \ t_1 + \xi \ r_1 \ t_1^2 - \xi \ r_\infty \ t_1^2 \right) \times t_i}{\frac{1}{r_0} + \Omega \ t + \xi \ t^2} + \xi \ r_\infty \ t^2
\]  

(2-1)

The variables \( \Omega \) and \( \xi \) are shaping parameters. From the 2008 data, the rates are as follows:
\[ r_0 = 0.0309 \]
\[ r_2 = 0.0288 \]
\[ r_{\infty} = 0.0435 \]

The resulting term structure curve is shown in Figure 2-2.

![Figure 2-2: Term Structure of Interest Rates Model](image)

### 2.3 Cost of Debt Capital

The cost of debt to the firm is based upon interest rates and a required interest spread due to credit and default risk (credit spread). Credit risk is dependent upon many different characteristics of the firm; however, the primary concern in this analysis is the partial effect of leverage and financial risk upon debt costs. For this analysis, credit spreads are estimated in two stages:

i. Calculation of expected credit spread- and by extension, cost of debt- based upon expected rating by Nationally Recognized Statistical Rating Organizations (NRSROs)

ii. Estimation of credit rating based upon financial risk and leverage

Figure 2-3 plots credit spreads for seven different indices over the period from 1998 through 2008. The corresponding summary statistics are shown in Figure 2-4 and Figure 2-5.
Summary Statistics for Corporate Credit Spreads, 1998 - 2008

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>68.48</td>
<td>82.54</td>
<td>102.71</td>
<td>146.83</td>
<td>322.59</td>
<td>711.79</td>
<td>1,319.87</td>
</tr>
<tr>
<td>Std Dev</td>
<td>19.65</td>
<td>24.94</td>
<td>34.68</td>
<td>57.58</td>
<td>145.65</td>
<td>476.88</td>
<td>656.43</td>
</tr>
<tr>
<td>Minimum</td>
<td>42.17</td>
<td>48.93</td>
<td>56.09</td>
<td>72.40</td>
<td>128.87</td>
<td>231.26</td>
<td>389.33</td>
</tr>
<tr>
<td>Maximum</td>
<td>129.51</td>
<td>153.83</td>
<td>181.21</td>
<td>303.30</td>
<td>806.75</td>
<td>1,934.66</td>
<td>2,608.47</td>
</tr>
<tr>
<td>Skew</td>
<td>-0.33</td>
<td>-0.57</td>
<td>-0.94</td>
<td>-0.53</td>
<td>1.05</td>
<td>-0.39</td>
<td>-0.93</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.33</td>
<td>-0.57</td>
<td>-0.94</td>
<td>-0.53</td>
<td>1.05</td>
<td>-0.39</td>
<td>-0.93</td>
</tr>
</tbody>
</table>

The following table describes the criteria used by Standard and Poor’s in their evaluation of financial risk:
Cost of Capital and Corporate Refinancing Strategy: Optimization of Costs and Risks

### Financial Risk and Projected Rating

<table>
<thead>
<tr>
<th>Financial Risk</th>
<th>Leverage</th>
<th>Expected Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal</td>
<td>Less than 0.25</td>
<td>AAA to BB</td>
</tr>
<tr>
<td>Modest</td>
<td>0.25 to 0.35</td>
<td>AA to B+</td>
</tr>
<tr>
<td>Intermediate</td>
<td>0.35 to 0.45</td>
<td>A to B+</td>
</tr>
<tr>
<td>Aggressive</td>
<td>0.45 to 0.55</td>
<td>BBB to B</td>
</tr>
<tr>
<td>Highly Leveraged</td>
<td>Greater than 0.55</td>
<td>BB to B-</td>
</tr>
</tbody>
</table>

**Figure 2-6: Standard and Poor's Financial Risk Criteria**

Using the assumption that corporations at the upper end of the rating range will generally exhibit leverage characteristics in the lower end of the leverage range, we are left with the following data:

<table>
<thead>
<tr>
<th>Rating</th>
<th>Leverage</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>1 SD Below Mean</th>
<th>1 SD Above Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.15</td>
<td>68.48</td>
<td>19.65</td>
<td>48.82</td>
<td>88.13</td>
</tr>
<tr>
<td>AA</td>
<td>0.25</td>
<td>82.54</td>
<td>24.94</td>
<td>57.60</td>
<td>107.48</td>
</tr>
<tr>
<td>A</td>
<td>0.35</td>
<td>102.71</td>
<td>34.68</td>
<td>68.03</td>
<td>137.39</td>
</tr>
<tr>
<td>BBB</td>
<td>0.45</td>
<td>146.83</td>
<td>57.58</td>
<td>89.25</td>
<td>204.41</td>
</tr>
<tr>
<td>BB</td>
<td>0.55</td>
<td>322.59</td>
<td>145.65</td>
<td>176.94</td>
<td>468.24</td>
</tr>
<tr>
<td>B</td>
<td>0.65</td>
<td>711.79</td>
<td>476.88</td>
<td>234.92</td>
<td>1,188.67</td>
</tr>
<tr>
<td>CCC</td>
<td>0.75</td>
<td>1,319.87</td>
<td>656.43</td>
<td>663.44</td>
<td>1,976.30</td>
</tr>
</tbody>
</table>

**Figure 2-7: Financial Risk and Expected Leverage**

It is clear from Figure 2-4 that the relationship between spread and expected leverage is nonlinear and exhibits heteroskedasticity. A scatter plot of the two variables further confirms this:

Therefore, before generating a regression, the following transformations are required:

\[\text{spread} \rightarrow \ln(\text{spread})\]

\[\text{leverage} \rightarrow \text{leverage}^2\]

By making these transformations, the variance is stabilized and a linear relationship is generated. The results may be tested with a revised scatter plot:
An ordinary least squares regression yields the following results:

Figure 2-9: Squared Leverage vs Log Spread
### Regression Output

**Regression Statistics**

| Multiple R | 0.9954 |
| R Square   | 0.9909 |
| Adjusted R Square | 0.9891 |
| Standard Error | 0.1192 |
| Observations | 7 |

**ANOVA**

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.7311</td>
<td>7.7311</td>
<td>544.4727</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>0.0710</td>
<td>0.0142</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7.8021</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Coefficients**

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
<th>Lower 95.0%</th>
<th>Upper 95.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>4.0083</td>
<td>0.0747</td>
<td>53.6658</td>
<td>0.0000</td>
<td>3.8163</td>
<td>4.2003</td>
<td>3.8163</td>
</tr>
<tr>
<td>leverage2</td>
<td>5.7333</td>
<td>0.2457</td>
<td>23.3339</td>
<td>0.0000</td>
<td>5.1016</td>
<td>6.3649</td>
<td>5.1016</td>
</tr>
</tbody>
</table>

### Residual Output

<table>
<thead>
<tr>
<th>Observation</th>
<th>Predicted ln(spread)</th>
<th>Residuals</th>
<th>Standard Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.1373</td>
<td>0.0892</td>
<td>0.8197</td>
</tr>
<tr>
<td>2</td>
<td>4.3666</td>
<td>0.0466</td>
<td>0.4288</td>
</tr>
<tr>
<td>3</td>
<td>4.7106</td>
<td>-0.0787</td>
<td>-0.7239</td>
</tr>
<tr>
<td>4</td>
<td>5.1693</td>
<td>-1.1800</td>
<td>-1.6550</td>
</tr>
<tr>
<td>5</td>
<td>5.7426</td>
<td>0.0338</td>
<td>0.3104</td>
</tr>
<tr>
<td>6</td>
<td>6.4306</td>
<td>0.1372</td>
<td>1.2611</td>
</tr>
<tr>
<td>7</td>
<td>7.2333</td>
<td>-0.0480</td>
<td>-0.4411</td>
</tr>
</tbody>
</table>

### Probability Output

<table>
<thead>
<tr>
<th>Percentile</th>
<th>ln(spread)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1429</td>
<td>4.2265</td>
</tr>
<tr>
<td>21.4286</td>
<td>4.4133</td>
</tr>
<tr>
<td>35.7143</td>
<td>4.6319</td>
</tr>
<tr>
<td>50.0000</td>
<td>4.9893</td>
</tr>
<tr>
<td>64.2857</td>
<td>5.7764</td>
</tr>
<tr>
<td>78.5714</td>
<td>6.5678</td>
</tr>
<tr>
<td>92.8571</td>
<td>7.1853</td>
</tr>
</tbody>
</table>

**Figure 2-10**: Spread and Leverage Regression Results

**Figure 2-11**: Fitted Regression Line
The t-statistics, f-statistic, and $R^2$ generated from this regression suggest that the model is a good fit. Figure 2-11 displays a graphical representation of the regression. The regression equation becomes:

$$\ln\text{spread} = 5.7333 \text{ leverage}^2 + 4.0083$$  \hspace{1cm} (2-2)

Or, equivalently:

$$\text{spread} = e^{5.7333 \text{ leverage}^2+4.0083}$$  \hspace{1cm} (2-3)

This spread is translated into an after-tax cost of debt capital using the following equation:

$$k_d = (1 - t)(r_f + e^{5.7333 \text{ leverage}^2+4.0083})$$  \hspace{1cm} (2-4)

The resulting cost of debt function (under a 40 percent tax rate) is graphically displayed in Figure 2-12.

2.4 Cost of Equity Capital

One generally accepted method for evaluating equity returns is the Capital Asset Pricing Model (CAPM). This formula equates the required return on equity to the sum of a risk-free rate and a risk premium related to the market risk of the stock. The formula is as follows:

$$k_e = r_f + \beta(r_m - r_f)$$  \hspace{1cm} (2-5)

Beta, however, is sensitive to the firm’s leverage due to its effect on financial risk. To compensate for this additional risk, a leveraged beta is calculated based upon the ratio of leverage. The final result also reflects a discount by the corporate tax rate (Damodaran 1994).
The final formula for cost of equity incorporating the leverage of the firm becomes:

$$k_e = r_f + \beta \left( 1 + (1 - t) \frac{D}{E} \right) (r_m - r_f)$$  \hspace{1cm} (2-7)

To set the variables of this model, it is necessary to estimate the expectation for market and risk-free return. For this purpose, historic annual returns are analyzed for investments in the Dow Jones Industrial Average (estimate of the aggregate stock market) and in 3-month Treasury Bills (a near risk-free asset). The geometric mean returns over the period from 1982 through 2008 are 8.62% and 5.25%, respectively. Historical returns for both indices are shown in Figure 2-13.

The final functions for cost of equity and beta are shown in Figure 2-14.
2.5 Weighted Average Cost of Capital and Optimal Leverage

With the cost of debt and cost of equity, it is a simple task to calculate total cost of capital to the firm (weighted average cost of capital, or WACC). Recall Formula (1-1) for WACC:

$$ WACC = \frac{D}{D+E}k_d + \frac{E}{D+E}k_e $$

(1-1)

This equation can be further simplified by substituting the following definition of leverage ratio:

$$ l = \frac{D}{D+E} $$

(2-8)

$$ WACC = lk_d + (1-l)k_e $$

(2-9)

Since WACC is a simple weighted average of $k_d$ and $k_e$, the value at $l = 0$ is equal to $k_e$, and the value at $l = 1$ is equal to $k_d$. At first, WACC decreases as cheaper debt replaces equity; at some point, however, elevated leverage causes the cost of both equity and debt to increase substantially and WACC again begins to rise. The optimal leverage for the firm is at the minimum of Equation (2-9). Figure 2-15 shows WACC against $k_d$ and $k_e$.

![Figure 2-15: Weighted Average Cost of Capital Curve](image)

It is also possible to define a penalty function representing the opportunity cost of deviating from the optimal level. This penalty function is shown in Figure 2-16.
From this analysis, the minimum WACC (6.69%) occurs at a capital structure composed of 53% debt and 47% equity.
3 Evaluation of Long-Term Strategies

3.1 Overview

Much literature has been devoted to evaluating cost of capital and enterprise value, evaluating marginal tax shelter benefits, and defining optimal target debt ratios, yet empirical real-world studies still provide contradictory results (Graham and Harvey 2001). It seems that firms are generally in disagreement as in how to best finance the firm, as well as when and how to refinance and adjust leverage levels to reflect financing targets. In many cases, the complications and additional costs of implementing a complex analytical model make it impractical in reality.

In this paper, I intend to develop a strategy with only three variables required to implement: a target leverage ratio, a refinancing minimum boundary, and a refinancing maximum boundary. First, I analyze the costs of refinancing as a firm attempts to readjust its finances closer to the optimal level. I then develop an applied algorithm for forecasting the cash flows of the firm, and simulate the actual total costs of financing. In the final section of this paper, repeated simulations of random cash flows under different refinancing boundary conditions allow for conclusions to be made about the distributions of capital costs under each set of boundary conditions. Ultimately, I conclude with the leveraging strategies which optimize weighted average cost of capital over the long-term horizon of the firm.

3.2 Costs of Refinancing

Earlier in this paper, we defined the U-shaped continuous penalty function describing the cost of capital for the firm given its current leverage ratio; there exists an optimal leverage ratio $l^*$, and the penalty increases as $l$ deviates from this ratio. In addition to these continuous financing charges, however, the firm is subjected to a discrete refinancing transaction cost $\rho$. Thus, the total cost of capital for the firm is a sum of the firm’s financing costs along with its refinancing costs (Bagley and Yaari, 1996).

Unlike financing costs, which are charged continuously, refinancing is a lump sum which is paid at the time of a restructuring. These costs are generally negotiated and can vary widely based on the transaction; nevertheless, they often share much in common. Most fees are assigned in terms of a percentage of the deal. A typical fee structure follows what is called the Lehman or 5-4-3-2-1 formula (Rock, Rock, and Sikora, 1994). In this case, the fee is tiered based on the size of the deal.
For example, 5 percent may be taken from the first million dollars, 4 percent from the second million dollars, and so on. The ultimate result is that the effective cost of financing decreases and approaches the minimum tier as the transaction becomes infinitely large. Figure 3-1 shows the variable refinancing cost using the *Lehman formula*.

![Figure 3-1: Lehman (5-4-3-2-1) Formula Variable Refinancing Costs](image)

Modifications such as the *double Lehman formula* build upon this concept. Additionally, many arrangements also allow for a flat fixed fee even if the deal is never consummated. In the context of this paper, the basic *Lehman formula* is used to model refinancing costs.

### 3.3 Structure of the Firm

#### 3.3.1 Overview

The model centers on a view of the firm as a sum of ownership by the firm’s stakeholders, notably:

i. Equity holders which, in this model, are presumed to be holders of basic common stock

ii. Debt holders which, in this model, are presumed to be holders of basic corporate bonds with no embedded options or features

The value of the company is the sum of the equity holders’ and the debt holders’ stakes in the firm. Thus, the total enterprise value of the firm $V$ can be equated as:

$$ V = D + E $$

(3-1)

As time progresses, the firm generates positive and negative cash flows $c$ which effectively alter the firm’s value $V$. We can model the growth in $V$ due to these cash flows as a stochastic process with some dynamic growth drift term $\mu$ and a normally distributed error term which varies according to a geometric Wiener process with standard deviation $\sigma$. The value of the firm $V$ can thus be modeled:
\[ dV = \mu V dt + \sigma V d\omega \]  
\[ (3-2) \]

Figure 3-2 shows several random paths for the function described in Equation (3-2).

### 3.3.2 Modeling Growth in Cash Flows

The growth of the firm \( \mu \) must be modeled as a deterministic function of time. For the purposes of this paper I use a fitting of the second-order rational function similar to that used for term structure in Section 2.2. With the firm’s current growth rate, a rate at some point in the future, and an expected terminal rate, the function described in Equation (2-1) creates a continuous curve into the future. The estimates used in this analysis are:

\[
\begin{align*}
    r_0 &= 0.04 \\
    r_8 &= 0.08 \\
    r_\infty &= 0.00 \\
    \Omega &= 0.0 \\
    \xi &= 0.5
\end{align*}
\]

The resulting forward growth curve is plotted in Figure 3-3.
3.3.3 Modeling Financial Leverage

Since the obligations of debt are constant in relation to the firm’s cash flows, $D$ is constant with respect to changes in $V$ and all gains or losses are forwarded to equity holders. Hence, in the absence of dividend payments and refinancing transactions:

$$dD = 0$$  \hspace{1cm} (3-3)

and

$$dE = dV = \mu V dt + \sigma V d\omega$$  \hspace{1cm} (3-4)

Then, by Itô’s lemma, the firm’s leverage may be modeled as:

$$dl = l \left( \frac{1}{2} \sigma^2 - \mu \right) dt - \sigma l d\omega$$  \hspace{1cm} (3-5)

In Figure 3-4, one possible path for $V$ is plotted along with its corresponding leverage function. In this example, the firm does not refinance at any point in time.
3.3.4 Refinancing Behavior

Aggregate financing cost is dependent upon the stochastic leverage process defined in Equation (3-5) as well as the refinancing behavior of the firm over time. Given an optimal or predetermined target leverage ratio $l^*$, the firm must decide when and how to restructure its finances to reflect its target. In this paper, I test the effects of strategic boundary settings where, if leverage reaches the boundary, the firm immediately refinances an amount necessary to return to the target. Refinancing occurs in two situations:

i. If the leverage ratio $l$ reaches the lower boundary, the firm issues additional debt and uses the proceeds to buy back equity

ii. If the leverage ratio $l$ reaches the upper boundary, the firm issues additional equity and uses the proceeds to retire debt

In each case, the firm suffers the corresponding transaction costs for the refinancing. This process is illustrated in Figure 3-5:
The following figures illustrate one cash flow process under two different refinancing strategies. With the first strategy (Figure 3-6 and Figure 3-7) the average WACC is 7.04%; with the second strategy (Figure 3-8 and Figure 3-9) the average WACC is 7.35%. 

**Figure 3-5: Refinancing Behavior of the Firm**

Instantaneous penalty function

Target leverage

Firm decreases leverage by issuing additional equity and using the proceeds to retire debt

Firm increases leverage by issuing new debt and using the proceeds to buy back equity

**Figure 3-6: Simulated Firm Value under Refinancing Boundaries of 0.30 and 0.70**

**Figure 3-7: Simulated Leverage under Refinancing Boundaries of 0.30 and 0.70**

**Figure 3-8: Simulated Firm Value under Refinancing Boundaries of 0.10 and 0.80**

**Figure 3-9: Simulated Leverage under Refinancing Boundaries of 0.10 and 0.80**
3.4 Assessing Optimality

Given the variables of the model, cost of capital is affected by several factors, namely:

i. The firm’s penalty cost of deviating from its optimal leverage ratio
ii. The cost of returning to a target leverage by refinancing
iii. The variability and growth in the firm’s cash flows
iv. How the firm sets its refinancing boundaries

We have developed estimations or models for factors i, ii, and iii. The final factor, setting refinancing boundaries, is the only factor over which the firm has control, and will be the variable which this model seeks to assess.

Optimal financing is generally considered to be the strategy where WACC is minimized. Due to the uncertainty of the firm’s cash flows and its corresponding effect on capital structure, however, true costs of capital can only be expressed as a probability. This allows for several optimizations dependent on the objectives and risk tolerance of the firm.
4 Calibration and Simulation

4.1 Overview

In this analysis, a Monte Carlo simulation with 10,000 trials is used to assess the effects of various refinancing strategies. For each trial, a set of random cash flows is generated over 1,000 months. A simulation is run for each set of cash flows using every possible upper and lower boundary in 5 percent increments.

As discussed earlier in this paper, several key inputs are necessary for this simulation:

i. The initial conditions of the firm, namely its current equity and debt structure
ii. Refinancing cost information; that is, the cost of retiring debt and issuing new equity or issuing new debt and buying back equity (the *Lehman formula* is used in this analysis)
iii. Estimations about equity and debt financing costs at given amounts of leverage; this includes the WACC curve and penalty function derived earlier in this paper
iv. Forecasts of future earnings for the firm; the stochastic model described in Section 3.3 is used for this analysis
v. Risk-free interest rate estimations for equity and debt cost as well as present value calculations; the forward curve derived earlier in this paper is used for this analysis

4.2 Assumptions

For this test the theoretical firm starts with an arbitrary value of $1,000,000 financed by 53 percent debt, the optimal ratio derived earlier in this paper. Refinancing costs are assumed to follow the *Lehman formula* described in Section 3.2. Cash flows are generated using the growth curve introduced in Section 3.3.2 and a standard deviation of 0.20. Interest rates follow the curve derived in section 2.2. It is also assumed that the target leverage of the firm is at the point on the curve where WACC is minimized.

4.3 Simulation Process

Using the structure defined in Equation (3-1), random cash flows are simulated subject to the dynamic growth rate $\mu$ and a random draw with standard deviation $\sigma$. The model functions as a discrete time process under 1-month intervals. With each movement in firm value, Equation (3-5) defines a corresponding change to leverage. When the leverage ratio exceeds the upper limit or falls
below the lower limit, an amount is refinanced such that the firm returns to its target ratio. Each refinancing, however, results in additional transaction costs charged at the time of the refinancing.

Below, several figures show the output from one particular trial. Figure 4-3 plots the 12-month lagging average WACC throughout the simulation and Figure 4-4 plots the cumulative average WACC for the firm since the beginning of the simulation. Figure 4-5 plots the cumulative amount, in present value, incurred through financing. The average WACC for this particular trial (under the boundary set of 0.30 and 0.70) is 6.79%.
Cost of Capital and Corporate Refinancing Strategy: Optimization of Costs and Risks

Figure 4-3: Example Trial Displaying 1-Year Lagging Annualized Cost

Figure 4-4: Example Trial Displaying Cumulative Average Cost Since Beginning of Simulation

Figure 4-5: Example Trial Displaying Cumulative Financing Costs in Present Value
4.4 Results and Analysis

4.4.1 Overview

At the derived optimal leverage ratio of 53 percent, 90 different sets of boundaries exist at 5 percent intervals. With the 10,000 sets of unique free cash flows generated in this test, 900,000 unique sets of cash flows result from the analysis.

This analysis produces clear evidence of the drastic effect of refinancing behavior on the mean and the distribution of total financing costs. Figure 4-6 and Figure 4-7 illustrate the probability distributions under each of the 90 unique refinancing boundaries; Figure 4-8 displays the resulting summary statistics.

When evaluating the effects of refinancing strategies, the firm can focus on several factors, namely:
i. The expected cost of capital resulting from the strategy, as measured by the mean

ii. The relative uncertainty in expected cost of capital, as measured by standard deviation

iii. The risk of suffering unexpected and biased cost outcomes, as measured by the non-normalness and shape of the probability distribution

In this section, I analyze the effects of different strategies on each of these three factors.
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Figure 4-8: Descriptive Statistics for Average Cost
4.4.2 Optimization of the Mean

In this simulation, the minimum average cost of capital (6.82 percent) is achieved using boundaries of 0.30 and 0.65. Note that this is slightly higher than the static optimal level of 6.69 percent; this reflects additional necessary costs due to deviations in leverage as the firm generates positive and negative cash flows. The means range a full percentage, from 6.82 percent to 7.82 percent. If the firm were to use a passive strategy (very wide boundaries), average cost is expected to be 7.39 percent. Thus, management’s financing policy can benefit the firm up to 57 basis points over a passive strategy; this translates into an 8 percent reduction in total financing costs. Summary statistics for the distribution of means at different refinancing boundaries are shown in Figure 4-9 and Figure 4-10.

<table>
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<tr>
<td>Median</td>
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<tr>
<td>Standard Deviation</td>
<td>0.20%</td>
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<tr>
<td>Minimum</td>
<td>6.82%</td>
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<td>Maximum</td>
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<td>Range</td>
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Figure 4-9: Distribution of the Mean

As Figure 4-10 illustrates, the cost of capital distribution is not symmetric. This is driven by the shape of the penalty function: while the benefits reach a minimum at the optimal leverage ratio, the potential penalty costs approach infinity as leverage approaches 100 percent.

The optimal strategy takes into account the tradeoff between transaction costs and the penalty costs associated with operating under suboptimal leverage. Two factors drive this strategy:
i. The penalty cost of erring on the side of greater leverage is much more expensive than its alternative.

ii. The growth drift of the firm is assumed to be positive throughout most of the firm’s life. Because of this, the lower boundary is more likely to be hit than the upper boundary.

Because of the first point, the firm is typically benefited by setting a low upper boundary. The high costs associated with inflated leverage ratios can have a disastrous effect on the firm’s ability to raise capital, and can easily outweigh the transaction costs necessary to refinance. This circumstance is characteristic of an unprofitable company as it builds up successive negative cash flows.

The second point addresses the growth characteristics of the firm. In its natural state, the firm tends to de-lever itself as positive cash flows build equity value. Thus, hitting the lower refinancing boundary is to be expected periodically in the course of operations for a profitable firm. Additionally, the costs to a firm with leverage below the optimal level are much more manageable than those for a firm above the optimal level; when deciding upon a lower refinancing boundary, the firm is allowed more flexibility in the ability to balance the transaction costs with the potential benefit of adjusting to a more optimal capital structure.

Figure 4-11 and Figure 4-12 aggregate both independent variables against mean cost. It is apparent that divergence in either direction from the optimal boundary set increases the penalty cost. The worst outcome occurs with a very high upper boundary (failing to avoid the extremely high capital costs associated with elevated leverage) and a very high lower boundary (subjecting the firm to transaction costs from unnecessary refinancing); in fact, this is the only active financing strategy with an expected average cost higher than the cost using a passive strategy. The benefit function is shown in Figure 4-13.
4.4.3 Analysis of Variance

In the previous section, expected average cost of capital was optimized for upper and lower boundary conditions. This section will assess the variability in outcomes dependent upon the boundary-setting strategies.

As Figure 4-7 showed, the probability distributions under different refinancing strategies vary widely not only in mean but also in dispersion. The figures below show the distributions for different refinancing strategies. Depending on the refinancing strategy, the distribution of outcomes can range from approximately normal (Figure 4-19) to heavily skewed (Figure 4-20), and narrowly
defined (Figure 4-14) to widely dispersed (Figure 4-15). The optimization, therefore, must also take into account the uncertainty of the prediction.

Dispersion of cost outcomes is high under two situations:

i. A boundary is set very close to the target; in this case refinancing costs, which are very sensitive to the path of the firm’s cash flows, drive the outcome.

ii. A boundary is set very far from the target; in this highly volatile cash flows subject the firm to high penalty costs as it remains at inefficient leverage ratios for extended amounts of time.

Consider a firm with no refinancing policy, or boundaries set very far from the target. From this analysis, the resulting cost of capital deviates an average of 0.21 percent from the mean. By comparison, the refinancing strategy with the least cost variability (0.20 and 0.55) results in a standard deviation of 0.05 percent. Thus, the firm can reduce its variability in costs by up to 74 percent by setting refinancing boundaries.

The optimal setting of the boundary depends on the penalty function as well as the leverage function of the firm. For the upper boundary, two factors drive the firm’s cost:

i. A volatile penalty function, where costs increase exponentially as leverage approaches 100 percent

ii. A low frequency of hits, since a growing firm tends to naturally de-lever itself

In this case, the extreme penalty costs of financing at a high leverage ratio outweigh the transaction costs associated with excessive refinancing. The firm can reduce the volatility in its cost of capital by setting low upper boundaries.

The lower leverage boundary is defined by two characteristics:
i. A moderate penalty function as compared to deviations on the highly leveraged side

ii. A high frequency of hits as the growing firm builds equity relative to its stable debt

Because of these factors, penalty costs are less sensitive to the setting of the lower boundary and much of the volatility is dependent upon excessive refinancing costs. The result is that variability is reduced by setting a lower boundary closer to zero.

The minimum standard deviation generated from this experiment, 0.05 percent, resulted from a lower boundary of 0.20 and an upper boundary of 0.55 (the lowest tested setting for upper boundary). Complete results are shown in Figure 4-16 and Figure 4-17.
4.4.4 Evaluating Efficiency

The Sharpe ratio is commonly used to assess efficiency in investment portfolio management. Essentially, it measures marginal rewards against risk. In the case of investments, the marginal reward is excess return or risk premium; for this purpose, I evaluate reduction in financing cost. Equation (4-1) defines the Sharpe ratio (Sharpe 1994).

\[
S = \frac{R - R_f}{\sigma} = \frac{E[R - R_f]}{\sqrt{\text{var}[R - R_f]}}
\]  

\(4-1\)

In the above equation, \(R\) is the cost of capital using a refinancing strategy, \(R_f\) is the cost of capital using a passive strategy, \(E[R - R_f]\) is the expected value of the cost reduction, and \(\sigma\) is the standard deviation of the cost reduction. In previous sections, mean and variance were found to be optimized with low upper boundaries and moderate lower boundaries. Variability left the opportunity for the greatest benefit; thus, the Sharpe ratio, which is driven by the underlying variance, shows a clear skew toward a low upper boundary. The maximization Sharpe ratio (9.60) occurs with boundaries of 0.25 and 0.55; the most efficient combination, therefore, produces 9.60 percent in cost reduction per unit of standard deviation. The high magnitude of this calculation is a strong advocate for the use of active refinancing. Sharpe ratios are displayed in Figure 4-18.

![Figure 4-18: Sharpe Ratio](image)

4.4.5 Minimizing Tail Risk

In this section, I evaluate three risk factors relating to the distribution of potential cost outcomes:
i. The bias or skew of the outcome distribution

ii. The peakedness or kurtosis of the distribution

iii. The value-at-risk measure, or the highest possible cost incurred at a 1 percent level of significance

Depending on the refinancing strategy used, a firm can expect unbiased, negatively skewed, or positively skewed cost distributions. Managers are interested in avoiding positively skewed outcomes because these signify a high tail risk; while the majority of outcomes will appear to be below the mean, distributions with a high positive skew also have the potential risk of experiencing extremely high unprecedented costs. Likewise, distributions with a low negative skew would tend experience costs slightly above the mean in most instances, but also have the potential to experience extremely low cost outliers. The most negative skew produced in this analysis (-0.96) results from boundaries of 0.50 and 0.55. In this case, the mean (7.26 percent) is rather high, but the potential exists for periods with extremely low costs. Two distributions- one with a near normal distribution and one with a heavy positive skew- are shown below. Measures of skew for each pair of boundaries are shown in Figure 4-21. Note that skew is greatest using passive strategies with wide boundaries; this suggests that the tail risk is high using a passive strategy, leaving much room for improvement by setting active refinancing boundaries.
Figure 4-21: Skew

Distributions with high kurtosis are often described as “fat tailed” because of the relative thickness on the tail ends of the distribution. In practice, this implies that outcomes are widely distributed and the probability of witnessing an extreme event in either direction is high. Naturally, low kurtosis is preferred to avoid the uncertainty associated with these outliers. Through this optimization it is possible to produce a platykurtic (flat-topped) distribution with a kurtosis of -0.44 using boundaries of 0.40 and 0.75. Kurtosis at each boundary is shown in Figure 4-22. Again, note that the greatest kurtosis is generated from passive strategies with wide boundaries. This provides additional evidence that unpredictability can be reduced through active refinancing policy.

Figure 4-22: Kurtosis
The final measure, value-at-risk, evaluates worst-case outcome under a given level of significance. This analysis considers the VaR at 1 percent significance. The minimum VaR of 6.98 percent is achieved using boundaries of 0.30 and 0.55; thus, with these boundaries, we can say with 99 percent confidence that the firm will experience costs less than 6.98 percent. This is a drastic improvement over the passive refinancing strategy, where the corresponding VaR is 101 basis points higher. The most extreme outcomes, with costs up to 9 percent, result from very high upper boundaries combined with very high lower boundaries. Full VaR results are shown in Figure 4-23.
5 Conclusion

This paper assessed the risks and rewards resulting from various strategies used by a firm to finance its assets. The analysis suggests that setting simple refinancing boundaries can be highly beneficial to the firm. By carefully setting these boundaries, a corporation can reduce its expected financing costs up to 8 percent. The greatest benefit, however, lies in the ability to reduce risk and uncertainty. For a firm using a passive refinancing strategy, the distribution of costs is highly non-normal and has a very high risk of extreme outcomes depending on the cash flows of the firm. I found that the firm can reduce its cost variability up to 74 percent relative to this passive financing strategy, and many of the key risks can be mitigated by constructing more normal cost distributions. Alternatively, the firm may decide to include but structure its cost risks, resulting in non-normal distributions aligning with the firm’s goals and risk tolerance.

Below is a summary of the resulting optimizations produced through this paper. By following one of these strategies, the firm can structure its financing policy to optimize its cost expectation, variability, or risk. Which strategy the firm ultimately chooses depends upon its objectives.
Since the variables of this model are expressed in percentages, the results are applicable for a firm of any size. The firm should, however, pay special attention to additional influences and violated assumptions before adopting a strategy given above. Some examples include:

i. A mature firm, with a declining growth curve, may choose to tighten its lower boundary to reflect its slower de-leveraging process

ii. A smaller firm may be subjected to higher (as a percentage) refinancing costs; this may induce the firm to widen both refinancing boundaries.

iii. A firm, for political reasons, may be restricted from allowing its leverage to reach a certain level, even if it would be mathematically optimal

iv. Corporate governance and competitive risk may induce a firm to restrict issuance of common stock in order to maintain control of the firm

v. Market conditions may provide opportunities for debt or equity issuance at attractive relative levels, regardless of the current capital structure of the firm

vi. Investors may look unfavorably at debt restructurings as leading indicators on the strength of the firm or management’s outlook

Financing costs can vary considerably depending on the strategy used by the firm. While the exact strategy may vary, this paper has found that the firm can better achieve its goals by setting active refinancing policies. Additionally, due to transaction costs, the firm must consider not only its costs at a single point in time but also the implications over the long term time horizon. Lastly, the firm need not set up complex and resource-devouring dynamic analyses to realize the benefit of capital structure optimization; a simple upper and lower leverage boundary may be all that is needed to provide extensive cost savings and risk reduction.
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7.3 Appendix C: Rational Function Curve Derivation

The following derivation is presented in *Mathematica* format. It begins with a standard rational function:

\[
\text{standardRationalFunction}[a, b, c, d, e, f, x] := \frac{a + b x + c x^2}{d + e x + f x^2}
\]

Since this is redundant, the expression can be simplified with a change of variables.

\[
\text{rationalFunction}[x, b, c, d, e, f] := \frac{1 + b x + c x^2}{d + e x + f x^2}
\]

We set conditions on this function for three known points:

\[
\text{rationalFunction}[x] = \begin{cases} 
  r_0, & x = 0 \\
  r_i, & x = t_i \\
  r_\infty, & x = \infty
\end{cases}
\]

Solving for these three sets of points results in values of the variables \(b, c, d\):

\[
\text{Conditions} = \text{Solve} \left[ \text{rationalFunction}[0, b, c, d, e, f] = r_0, \text{rationalFunction}[t_i, b, c, d, e, f] = r_i \right.
\]

Collecting the results subject to the defined conditions for \(r_0, r_i, r_\infty\) results in a curve which should hit all three points.

\[
\text{result} = \text{Collect}[\text{rationalFunction}[x, b, c, d, e, f] /. \text{conditions}, \{r_0, r_i, r_\infty\}, \text{FullSimplify}]\
\]
Thus, the fitted function is:

\[
growthRate[x, r_0, r_i, r_\infty, t_i, d, e, f] = 1 + \frac{\left(-1 + \frac{r_i}{r_0} + e r_i t_i + f r_i t_i^2 - f r_\infty t_i^2\right) x}{t_1} + f r_\infty t^2 + \frac{1}{r_0} + e x + f x^2
\]

Checking the conditions at \(x\) - values of \(t_0, t_i, \) and \(t_\infty\) results in the expected values of \(r_0, r_i,\) and \(r_\infty:\)

\[
\text{FullSimplify}[growthRate[t_i, r_0, r_\infty, r_i, t_i, d, e, f]]
\]

\[
r_i
\]

\[
\text{FullSimplify}[growthRate[0, r_0, r_\infty, r_i, t_i, d, e, f]]
\]

\[
r_0
\]

\[
\lim_{x \to \infty} growthRate[x, r_0, r_\infty, r_i, t_i, d, e, f]
\]

\[
r_\infty
\]

Since the variable \(d\) is not present in the resulting formula, it can be omitted in the final equation.
To maintain the notation of using Greek characters as shaping parameters, the shaping parameters \(e\) and \(f\) are replaced by \(\Omega\) and \(\xi\), respectively. The variable \(x\) may also be more accurately defined as \(t:\)

\[
growthRateFinal[t, r_0, r_i, r_\infty, t_i, \Omega, \xi] = 1 + \frac{\left(-1 + \frac{r_i}{r_0} + \Omega r_i t_i + \xi r_i t_i^2 - \xi r_\infty t_i^2\right) x}{t_i} + \frac{1}{r_0} + \Omega t + \xi t^2
\]

A sample plot returns a reasonable curve, hitting all defined points:

\[
\text{Plot}[growthRateFinal[t, .05, 0, 1,5,20,1], \{t, 0,80\}, PlotRange\{0,.12\}]
\]
7.4 Appendix D: Additional Mathematical Definitions

Skewness

\[
\text{skewness} = \frac{\sum_{i=1}^{N} (Y_i - \bar{Y})^3}{(N - 1)s^3}
\] (7-1)

Kurtosis

\[
kurtosis = \frac{\sum_{i=1}^{N} (Y_i - \bar{Y})^4}{(N - 1)s^4}
\] (7-2)

Itō's Lemma

For an Itō drift-diffusion process:

\[
dX_t = \mu_t dt + \sigma_t d\omega_t
\] (7-3)

where \(dB_t\) is a differential of Brownian motion, and a twice continuously differentiable function:

\[
f(t, x)
\] (7-4)

Then:

\[
df(t, X_t) = \left( \frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2 f}{\partial x^2} \right) dt + \frac{\partial f}{\partial x} \sigma_t d\omega_t
\] (7-5)